# Similarity 

Geometry
Chapter 6

## Geometry 6

- This Slideshow was developed to accompany the textbook
- Larson Geometry
- By Larson, R., Boswell, L., Kanold, T. D., \& Stiff, L. 2011 Holt McDougal
- Some examples and diagrams are taken from the textbook.


### 6.1 Ratios, Proportions, and the Geometric Mean Ratio

- Comparing one number to another.
- Written as 1:64 or $\frac{1}{64}$
- My toy tractors are 1:64 scale. That means that one inch on the model is 64 inches on the real thing.
- Simplify the ratio

24 yards to 3 yards


$$
\frac{24}{3}=\frac{8}{1}
$$

# 6.1 Ratios, Proportions, and the <br> Geometric Mean <br> A triangle's angle measures are in the extended ratio of $1: 3: 5$. Find the measures of the angles 

$1 x+3 x+5 x=180$
$9 x=180$
$x=20$
Angles are 20, 60, 100

### 6.1 Ratios, Proportions, and the

 Geometric Mean
## Proportion

- Two ratios that are equal are a proportion.
- $\frac{1}{64}=\frac{2}{128}$ is a proportion
- ${ }^{64}$ The ${ }^{128}$ cross products of a proportion are equal.
- The height of my toy tractor is 1.5 inches, what is the height of the real tractor?
- $\frac{1}{64}=\frac{1.5}{x}$
- $1 \mathrm{x}={ }^{x} 64(1.5) \rightarrow \mathrm{x}=96$
- The real tractor is 96 inches tall (8 feet)


# 6.1 Ratios, Proportions, and the Geometric Mean 

, Find value of $x$.
$\frac{x-2}{x}=\frac{3}{8}$

$$
\begin{gathered}
\frac{x-2}{x}=\frac{3}{8} \\
8(x-2)=3 x \\
8 x-16=3 x \\
5 x=16 \\
x=\frac{16}{5}
\end{gathered}
$$

# 6.1 Ratios, Proportions, and the Geometric Mean Geometric Mean 

The geometric mean of two positive numbers a and b is the positive number x that satisfies $\frac{a}{x}=\frac{x}{b}$. So, $x=\sqrt{a b}$

Find the geometric mean of 18 and 54.

$$
\begin{gathered}
x=\sqrt{18 \cdot 54} \\
x=\sqrt{972} \\
x=31.18
\end{gathered}
$$

## Answers and Quiz

- 6.1 Answers
- 6.1 Quiz


### 6.2 Use Proportions to Solve Geometry Problems

- Properties of Proportions

$$
\frac{2}{3}=\frac{4}{6}
$$

- Cross products are equal
- $2 \cdot 6=3 \cdot 4$
- Reciprocals are equal
- $\frac{3}{2}=\frac{6}{4}$
- Can interchange numbers along a diagonal
- $\frac{2}{4}=\frac{3}{6}$
- Can add the denominator to the numerator
- $\frac{2+3}{3}=\frac{4+6}{6}$


### 6.2 Use Proportions to Solve Geometry Problems

$\frac{M N}{R S}=\frac{N P}{S T}$. Find x .


- $\frac{D E}{A C}=\frac{B E}{B C}$, find AC.


$$
\begin{gathered}
\frac{M N}{R S}=\frac{N P}{S T} \\
\frac{8}{10}=\frac{4}{x} \\
8 x=4 \cdot 10 \\
8 x=40 \\
x=5
\end{gathered}
$$

$$
\begin{gathered}
\frac{D E}{A C}=\frac{B E}{B C} \\
\frac{12}{A C}=\frac{18}{18+6} \\
12 \cdot 24=18 \cdot A C \\
288=18 \cdot A C \\
16=A C
\end{gathered}
$$

### 6.2 Use Proportions to Solve Geometry Problems

- Two cities are 96 miles from each other. The cities are 4 inches apart on a map. Find the scale of the map.

- 367 \#2-18 even, 22-28 even, 38, $39=15$

Extra Credit 370 \#2, $6=+2$

$$
\frac{4 \text { inches }}{96 \text { miles }}=\frac{1 \text { inch }}{24 \text { miles }}
$$

## Answers and Quiz

- 6.2 Answers
- 6.2 Quiz


### 6.3 Use Similar Polygons

- When I put something on the overhead projector, the projected image is larger than what is on the screen. The image is of a different size, but the same shape as what I write. They are similar.

Have the overhead ready

### 6.3 Use Similar Polygons

## Similar figures

- When two figures are the same shape but different sizes, they are similar.


## Similar polygons (~)


, Polygons are similar iff corresponding angles are congruent and corresponding sides are proportional.

Draw polygon on projector. Have student measure angles on the projector and the screen. Have another student measure the sides on the projector and on the screen. Check to see if corresponding angles are congruent. Check to see if corresponding sides are proportional.

### 6.3 Use Similar Polygons

- Ratio of lengths of corresponding sides is the scale factor.

ABCD ~ QRST

- What is the scale factor of QRST to ABCD?

Find $x$.


Scale factor: $\frac{6}{12}=\frac{1}{2}$
$\mathrm{x}: \frac{1}{2}=\frac{4}{x} \rightarrow \mathrm{x}=8$

### 6.3 Use Similar Polygons

Perimeters of Similar Polygons
If two polygons are similar, then the ratio of their perimeters is equal to the ratios of their corresponding side lengths.
, Congruent polygons have a scale factor of 1:1

### 6.3 Use Similar Polygons

ABCDE ~ FGHJK
Find the scale factor of FGHJK to ABCDE

- Find $x$

- Find the perimeter of $A B C D E$

Scale factor: $\frac{15}{10}=\frac{3}{2}$
$\mathrm{x}: \frac{3}{2}=\frac{18}{x} \rightarrow 3 x=36 \rightarrow x=12$
Perimeter: $\frac{3}{2}=\frac{15+9+12+15+18}{P} \rightarrow \frac{3}{2}=\frac{69}{P} \rightarrow 3 P=138 \rightarrow \mathrm{P}=46$

### 6.3 Use Similar Polygons

- $\Delta \mathrm{JKL} \sim \Delta E F G$. Find the length of the median $\overline{K M}$.

- 376 \#2-24 even, 28, 32-48 even $=22$

$$
\begin{gathered}
\frac{48}{40}=\frac{K M}{35} \\
40 \cdot K M=48 \cdot 35 \\
40 \cdot K M=1680 \\
K M=42
\end{gathered}
$$

## Answers and Quiz

- 6.3 Answers
- 6.3 Quiz


### 6.4 Prove Triangles Similar by AA

- Draw two triangles with two pairs of congruent angles. Measure the corresponding sides. Are they proportional? Are the triangles similar?


## AA Similarity

If two angles of one triangle are congruent to two angles of another triangle, then the triangles are similar.

### 6.4 Prove Triangles Similar by AA

- Show that the triangles are similar. Write a similarity statement.

$\Delta \mathrm{FGH} \sim \Delta \mathrm{QRS}$ by AA Similarity
$m \angle C D F=58$ by Triangle Sum Theorem
$\triangle C D F \sim \Delta D E F$ by AA Similarity


### 6.4 Prove Triangles Similar by AA

- You can use similar triangles to find things like the height of a tree by using shadows. You put a stick perpendicular to the ground Measure the stick and the shadow. Then measure the shadow of the tree. The triangles formed by the stick and the shadow and the tree and its shadow are similar so the height of the tree can be found by ratios. Suppose we use a meter stick. The stick's shadow is 3 m . The tree's shadow is 150 m . How high is the tree?
- 384 \#4-22 even, 26-32 even, 41-46 all, 48 = 21

$$
\begin{gathered}
\frac{\text { Tree Shadow }}{\text { Stick Shadow }}=\frac{\text { Tree Height }}{\text { Stick Height }} \\
\frac{150}{3}=\frac{x}{1} \\
x=50 \mathrm{~m}
\end{gathered}
$$

## Answers and Quiz

- 6.4 Answers
- 6.4 Quiz


# 6.5 Prove Triangles Similar by SSS and SAS 

SSS Similarity
It the measures of the corresponding sides of two triangles are proportional, then the triangles are similar.

## SAS Similarity <br> If the measures of two sides of a triangle are proportional to the measures of two corresponding sides of another triangle and the included angles are congruent, then the triangles are similar.

SSS Similarity - That's what happens when you enlarge a drawing.

### 6.5 Prove Triangles Similar by SSS

 and SAS- Which of the three trianales are similar?

- The shortest side of a triangle similar to $\triangle$ RST is 12 units long. Find the other side lengths of the triangle.

Try $\Delta \mathrm{LMN}$ and $\triangle \mathrm{RST}: \frac{20}{24}=\frac{24}{30}=\frac{26}{33}$ This is not true.
Try $\Delta \mathrm{LMN}$ and $\triangle \mathrm{XYZ}: \frac{20}{30}=\frac{24}{36}=\frac{26}{39}$ This is true. $\Delta \mathrm{LMN} \sim \Delta \mathrm{YZX}$
Try $\triangle \mathrm{XYZ}$ and $\triangle \mathrm{RST}: \frac{30}{24}=\frac{36}{30}=\frac{39}{33}$ This is not true.
$\frac{12}{24}=\frac{x}{30}=\frac{y}{33}$
$\frac{12}{24}=\frac{x}{30} \rightarrow 24 \mathrm{x}=360 \rightarrow \mathrm{x}=15$
$\frac{12}{24}=\frac{y}{33} \rightarrow 24 y=396 \rightarrow x=16.5$

### 6.5 Prove Triangles Similar by SSS

 and SAS, Explain how to show that the indicated triangles are similsr

- $\triangle$ SRT $\sim \Delta P N Q$

- $\Delta X Z W$ ~ $\Delta Y Z X$

6.5 Prove Triangles Similar by SSS and SAS
- 391 \#4-24 even, 25, 26, 30, 32, 36, 38, 4144 all $=21$
- Extra Credit 395 \#2, $6=+2$


## Answers and Quiz

- 6.5 Answers
- 6.5 Quiz


### 6.6 Use Proportionality Theorems

Triangle Proportionality Theorem
It a line is paraliel to a side of a triangle, then it separates the other two sides into proportional segments.


- And the converse is also true. Proportional segments $\rightarrow$ line parallel to the third side.


### 6.6 Use Proportionality Theorems

- Example:
- In $\triangle R S Q$ with chord $T U, Q R=10, Q T=2, U R=6$, and $S R=12$. Determine if $\overline{Q S} \| \overline{T U}$.


ANS: TR = 10-2 = 8, US = 12-6 = 6
TR/QT $=$ RU/US $\rightarrow 8 / 2=6 / 6 \rightarrow 4=1$ False, not parallel

### 6.6 Use Proportionality Theorems

If three or more parallel lines intersect two transversals, then they cut off the transversals proportionally.

## , Example:

- Using the information in the diagram, find the distance TV.
R

$$
\begin{gathered}
\frac{T U}{340}=\frac{160}{320} \\
320 T U=54400 \\
T U=170 \\
T V=T U+U V=170+340=510 \mathrm{~m}
\end{gathered}
$$

### 6.6 Use Proportionality Theorems

An angle bisector in a triangle separates the opposite side into segments that have the same ratio as the other two sides.

- Find $x$


400 \#2-18 even, 22, 24, 28, 30-36 even $=16$
$10 / x=12 /(18-x) \rightarrow 10(18-x)=12 x \rightarrow 180-10 x=12 x \rightarrow 180=22 x \rightarrow x=$ $180 / 22=8.18$

## Answers and Quiz

- 6.6 Answers
- 6.6 Quiz


### 6.7 Perform Similarity Transformations

- Dilation

- Transformation that stretches or shrinks a figure to create a similar figure.
- The figure is enlarged or reduced with respect to a point called the center of dilation
- The scale factor of a dilation is the ratio of a side of the image to the corresponding side of the original


### 6.7 Perform Similarity

## Transformations

- Coordinate Notation for a Dilation with respect to the origin
- $(x, y) \rightarrow(k x, k y)$ where $k$ is the scale factor
- If $0<k<1$, reduction
- If $\mathrm{k}>1$, enlargement


### 6.7 Perform Similarity Transformations

, Find the coordinates of $L, M$, and $N$ so that $\Delta \mathrm{LMN}$ is a dilation of $\triangle \mathrm{PQR}$ with a scale factor of $k$. Write the coordinate notation for the dilation.

- $P(-2,-1), Q(-1,0), R(0,-1) ; k=4$
$(x, y) \rightarrow(4 x, 4 y)$
$\mathrm{L}(-8,-4), \mathrm{M}(-4,0), \mathrm{N}(0,-4)$


### 6.7 Perform Similarity

## Transformations

- Suppose a figure containing the origin is dilated. Explain why the corresponding point in the image of the figure is also the origin.
, Origin is $(0,0) . k(0)=0$. so $(k 0, k 0)=(0,0)$
- 412 \#2-22 even, 30, 36, 38, 40-43 all $=18$
- Extra Credit 415 \#2,4 = +2
$(\mathrm{x}, \mathrm{y}) \rightarrow(\mathrm{kx}, \mathrm{ky})$
If x and y are 0 , then kx and ky are 0 .


## Answers and Quiz

- 6.7 Answers
- 6.7 Quiz


## 6.Review

- 422 \#1-16 = 16


## CAIPIITMEST

## Solve the propertion.

1. $\frac{6}{x}=\frac{9}{24} \quad$ 2. $\frac{3}{4}-\frac{y-5}{12}$
2. $\frac{3-7 t}{4}=\frac{3}{2}$
3. $\frac{7}{2 a+8}=\frac{1}{a-1}$

In Kxercises 5-7, use the diagram where $\triangle P Q R$ - $\triangle A B C$.
3. List all pairs of congruent angles.
6. Write the ratios of the corresp
statement of proportionality.
7. Find the value of $x$.


Determine whether the triangles are similar. If so, write a similarity
statement and the postulate or theorem that justifies your answer.
(20)



In Exercises 11-13, find the length of $\overline{\pi R}$.


12


Determine whether the dilation from Figure A to Figure $B$ is a reduction
or an enlargement. Then find its scale factor.
14.

15.

16. Scalk moon. You are making a scale model of your schoof s baseball diamond as part of an art project.
The distance between two consecutive bases is The distance between two consecutive bases is so feet. If you use a scale factor of $\frac{1}{180}$ to build your model. what will be the distance around the bases on your modelt

