Similarity

Geometry Chapter 6

Geometry 6

- This Slideshow was developed to accompany the textbook
 - Larson Geometry
 - By Larson, R., Boswell, L., Kanold, T. D., & Stiff, L.
 - 2011 Holt McDougal
- Some examples and diagrams are taken from the textbook.

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6.1 Ratios, Proportions, and the Geometric Mean

- Comparing one number to another.
- Written as 1:64 or $\frac{1}{64}$
- My toy tractors are 1:64 scale. That means that one inch on the model is 64 inches on the real thing.
- Simplify the ratio
 - 24 yards to 3 yards



 $\frac{24}{3} = \frac{8}{1}$

6.1 Ratios, Proportions, and the Geometric Mean A triangle's angle measures are in the extended ratio of 1 : 3 : 5. Find the measures of the angles

1x + 3x + 5x = 180 9x = 180 x = 20Angles are 20, 60, 100

6.1 Ratios, Proportions, and the Geometric Mean Proportion

- Two ratios that are equal are a proportion.
- $\frac{1}{64} = \frac{2}{128}$ is a proportion
- The cross products of a proportion are equal.
- The height of my toy tractor is 1.5 inches, what is the height of the real tractor?
 - ${}^{64}_{1x} = {}^{x}_{64}(1.5) \rightarrow x = 96$
 - The real tractor is 96 inches tall (8 feet)



$$\frac{x-2}{x} = \frac{3}{8}$$
$$8(x-2) = 3x$$
$$8x - 16 = 3x$$
$$5x = 16$$
$$x = \frac{16}{5}$$

6.1 Ratios, Proportions, and the Geometric Mean Geometric Mean

The geometric mean of two positive numbers a and b is the positive number x that satisfies $\frac{a}{x} = \frac{x}{b}$. So, $x = \sqrt{ab}$

Find the geometric mean of 18 and 54.

▶ 360 #4-44 even, 50, 52, 60, 72-80 even = 29

 $x = \sqrt{18 \cdot 54}$ $x = \sqrt{972}$ x = 31.18

Answers and Quiz • <u>6.1 Answers</u> • <u>6.1 Quiz</u>





$$\frac{MN}{RS} = \frac{NP}{ST}$$
$$\frac{8}{10} = \frac{4}{x}$$
$$8x = 4 \cdot 10$$
$$8x = 40$$
$$x = 5$$
$$\frac{DE}{AC} = \frac{BE}{BC}$$
$$\frac{12}{AC} = \frac{18}{18 + 6}$$
$$12 \cdot 24 = 18 \cdot AC$$
$$288 = 18 \cdot AC$$
$$16 = AC$$



 $\frac{4 \text{ inches}}{96 \text{ miles}} = \frac{1 \text{ inch}}{24 \text{ miles}}$

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6.3 Use Similar Polygons

When I put something on the overhead projector, the projected image is larger than what is on the screen. The image is of a different size, but the same shape as what I write. They are similar.

Have the overhead ready



Draw polygon on projector. Have student measure angles on the projector and the screen. Have another student measure the sides on the projector and on the screen. Check to see if corresponding angles are congruent. Check to see if corresponding sides are proportional.



Scale factor: $\frac{6}{12} = \frac{1}{2}$ x: $\frac{1}{2} = \frac{4}{x} \rightarrow x = 8$

6.3 Use Similar Polygons

Perimeters of Similar Polygons If two polygons are similar, then the ratio of their perimeters is equal to the ratios of their corresponding side lengths.

Congruent polygons have a scale factor of 1:1



Scale factor: $\frac{15}{10} = \frac{3}{2}$ x: $\frac{3}{2} = \frac{18}{x} \rightarrow 3x = 36 \rightarrow x = 12$ Perimeter: $\frac{3}{2} = \frac{15+9+12+15+18}{P} \rightarrow \frac{3}{2} = \frac{69}{P} \rightarrow 3P = 138 \rightarrow P = 46$



 $\frac{48}{40} = \frac{KM}{35}$ $40 \cdot KM = 48 \cdot 35$ $40 \cdot KM = 1680$ KM = 42

Answers and Quiz • <u>6.3 Answers</u> • <u>6.3 Quiz</u>





 Δ FGH ~ Δ QRS by AA Similarity

m \angle CDF = 58 by Triangle Sum Theorem \triangle CDF ~ \triangle DEF by AA Similarity



 $\frac{\text{Tree Shadow}}{\text{Stick Shadow}} = \frac{\text{Tree Height}}{\text{Stick Height}}$ $\frac{150}{3} = \frac{x}{1}$ x = 50 m

Answers and Quiz • <u>6.4 Answers</u> • <u>6.4 Quiz</u>

6.5 Prove Triangles Similar by SSS and SAS

SSS Similarity

If the measures of the corresponding sides of two triangles are proportional, then the triangles are similar.

SAS Similarity

If the measures of two sides of a triangle are proportional to the measures of two corresponding sides of another triangle and the included angles are congruent, then the triangles are similar.

SSS Similarity - That's what happens when you enlarge a drawing.



Try Δ LMN and Δ RST: $\frac{20}{24} = \frac{24}{30} = \frac{26}{33}$ This is not true. Try Δ LMN and Δ XYZ: $\frac{20}{30} = \frac{24}{36} = \frac{26}{39}$ This is true. Δ LMN ~ Δ YZX Try Δ XYZ and Δ RST: $\frac{30}{24} = \frac{36}{30} = \frac{39}{33}$ This is not true. $\frac{12}{24} = \frac{x}{30} = \frac{y}{33}$ $\frac{12}{24} = \frac{x}{30} \Rightarrow 24x = 360 \Rightarrow x = 15$ $\frac{12}{24} = \frac{y}{33} \Rightarrow 24y = 396 \Rightarrow x = 16.5$



6.5 Prove Triangles Similar by SSS and SAS

- 391 #4-24 even, 25, 26, 30, 32, 36, 38, 41-44 all = 21
- *Extra Credit 395 #2, 6 = +2*

Answers and Quiz • <u>6.5 Answers</u> • <u>6.5 Quiz</u>

6.6 Use Proportionality Theorems

Triangle Proportionality Theorem If a line is parallel to a side of a triangle, then it separates the other two sides into proportional segments. q = T

And the converse is also true. Proportional segments → line parallel to the third side.

S

U

6.6 Use Proportionality Theorems



• In \triangle RSQ with chord TU, QR = 10, QT = 2, UR = 6, and SR = 12. Determine if $\overline{QS} \parallel \overline{TU}$.



ANS: TR = 10 - 2 = 8, US = 12 - 6 = 6TR/QT = RU/US $\rightarrow 8/2 = 6/6 \rightarrow 4 = 1$ False, not parallel



$$\frac{TU}{340} = \frac{160}{320}$$
$$320TU = 54400$$
$$TU = 170$$
$$TV = TU + UV = 170 + 340 = 510 m$$



 $10 / x = 12 / (18 - x) \rightarrow 10 (18 - x) = 12x \rightarrow 180 - 10x = 12x \rightarrow 180 = 22x \rightarrow x = 180/22 = 8.18$

Answers and Quiz • <u>6.6 Answers</u> • <u>6.6 Quiz</u>

6.7 Perform Similarity Transformations

Dilation



- Transformation that stretches or shrinks a figure to create a similar figure.
- The figure is enlarged or reduced with respect to a point called the center of dilation
- The scale factor of a dilation is the ratio of a side of the image to the corresponding side of the original

6.7 Perform Similarity Transformations

- Coordinate Notation for a Dilation with respect to the origin
 - (x, y) \rightarrow (kx, ky) where k is the scale factor
 - \circ If 0 < k < 1, reduction
 - \circ If k > 1, enlargement

6.7 Perform Similarity Transformations

- Find the coordinates of L, M, and N so that ΔLMN is a dilation of ΔPQR with a scale factor of k. Write the coordinate notation for the dilation.
- P(-2, -1), Q(-1, 0), R(0, -1); k = 4

 $(x, y) \rightarrow (4x, 4y)$ L(-8, -4), M(-4, 0), N(0, -4)



 $(x, y) \rightarrow (kx, ky)$

If x and y are 0, then kx and ky are 0.

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